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# Influence of self-affine surface roughness on the friction coefficient for rubbers

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For a viscous rubber sliding onto a self-affine rough surface, the friction coefficient increases monotonically with decreasing roughness exponent  $H$  and at a rapid rate for  $H < 0.5$ . This is because the surface becomes rougher (with decreasing  $H$ ) at short roughness wavelengths ( $< \xi$ ) leading therefore to increased friction. Similar is also the behavior with decreasing the in-plane roughness correlation length  $\xi$  (for fixed roughness amplitude  $w$ ). Nevertheless, the roughness exponent  $H$  appears to influence more the friction coefficient than the in-plane correlation length  $\xi$ . For relatively low sliding speeds, analytic calculations of the coefficient friction were also feasible for the limiting roughness exponents  $H=0$  and  $H=1$ . Finally, under conditions of incomplete contact the friction coefficient was shown to decrease below its value of complete contact for contact lengths  $\lambda_{\text{con}} < \xi$  and relatively high sliding velocities. © 2003 American Institute of Physics. [DOI: 10.1063/1.1616985]

## I. INTRODUCTION

The phenomenon of sliding friction in between a rubber body sliding onto a hard solid substrate is a subject of fundamental and technological importance in the car industry (i.e., tire construction, rubber blades of wipers), cosmetic industry, etc.<sup>1–4</sup> The major difference in the frictional properties of rubbers with respect to other solids arises from their low elastic modulus and the high internal friction that is present over a wide range of frequencies.<sup>5</sup> Indeed, it was shown that the rubber friction is strongly related to its internal friction.<sup>2</sup>

In most cases sliding occurs on rough surfaces with a significant degree of randomness. Indeed, random rough surfaces, which are commonly encountered on solid surfaces,<sup>6,7</sup> possess roughness over various length scales rather than over a single one. This is a factor that has to be taken carefully into account in contact and friction related phenomena.<sup>5</sup> On the other hand, the friction force between a rubber and a hard rough solid substrate has two contributions which are called adhesive and hysteric.<sup>1</sup> The latter contribution arises from the oscillating forces that the surface asperities exert onto the rubber surface leading effectively to cyclic deformations and energy dissipation due to internal frictional damping.<sup>5</sup> As a result the hysteric contribution will have the same temperature dependence as that of an elastic modulus  $E(\omega)$ .<sup>5</sup> Finally, the adhesive component is important for clean and relatively smooth surfaces.<sup>5</sup>

Depending on the sliding velocity, the low elastic modulus of rubbers leads to instabilities. Indeed, at high sliding velocities (and for relatively smooth surfaces) the so-called Schallamach wave<sup>1</sup> appears. In this case, a compressed rubber surface in front of the contact area undergoes a buckling producing detachment waves from the front-end to the back-

end of the contact area. Here we will limit our study to low speeds to exclude this case.<sup>5</sup>

If we assume a rubber to slide with velocity  $V$  over a rough surface with lateral length scale  $L$  (i.e., a sinus periodic profile), then it will feel fluctuating forces with frequencies in the range  $\omega \approx V/L$ . If in addition, the surface has a wider distribution of length scales  $L$ , then a wider distribution of frequency components in the Fourier decomposition of the surface stresses acting on the sliding rubber will be also present.<sup>5</sup> Moreover, the contribution of surface roughness to friction coefficient  $\mu_f$  at length scales  $L$  is maximum for relaxation time  $\tau \approx L/V$  which is located in the transition region between rubber (low  $\omega$ ) and glass (high  $\omega$ ) behavior.<sup>5</sup>

So far it has been shown that for self-affine random rough surfaces, the coefficient of friction  $\mu_f$  depends significantly on the roughness exponent  $H$  ( $0 \leq H \leq 1$ ), which characterizes the degree of surface irregularity (as  $H$  becomes smaller the surface becomes more irregular at short length scales) at short length scales.<sup>6,7</sup> Nevertheless, the previous studies were performed using only power-law approximations for the self-affine roughness spectrum. This is valid for lateral roughness wavelengths  $q\xi > 1$  with  $\xi$  the in-plane roughness correlation length. This work concentrates on the effect of roughness, including the contributions from roughness wavelengths  $q\xi \leq 1$ .

## II. COEFFICIENT OF FRICTION UNDER COMPLETE CONTACT

Under conditions of complete contact between an elastic body of Young modulus  $E$  and Poisson ratio  $\nu$  with a solid rough surface, the coefficient of friction upon sliding with velocity  $V$  is given by<sup>5</sup>

$$\mu_f = \frac{1}{2} \int_0^{Q_c} q^3 C(q) dq \int_0^{2\pi} \text{Im} \left[ \frac{E^*(qV\tau \cos \phi)}{(1 - \nu^2)\sigma} \right] \cos \phi \, d\phi, \quad (1)$$

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where  $C(q)$  is the Fourier transform (power spectral density) of the autocorrelation function  $C(r) = \langle h(\mathbf{r})h(0) \rangle$  with  $h(\mathbf{r})$  the surface roughness height so that  $\langle h \rangle = 0$ . The symbol  $\langle \dots \rangle$  stands as an ensemble average over possible roughness configurations.  $\sigma$  is the applied load, and  $E^*(\omega)$  is the complex conjugate of the frequency dependent Young modulus  $E(\omega)$ . The latter is assumed to be given by the rheological model<sup>5</sup>

$$E(\omega) = \frac{E_1[(1+\alpha) + (\omega\tau)^2]}{(1+\alpha)^2 + (\omega\tau)^2} - j \frac{\alpha\omega\tau E_1}{(1+\alpha)^2 + (\omega\tau)^2}, \quad (2)$$

with  $E_1 = E(\infty)$ , and  $E(\infty)/E(0) = 1 + \alpha$  (with typical values for the parameter  $\alpha = 10^3$ ).<sup>5</sup> Although Eq. (2) gives a rather abrupt transition between the glassy and rubber region, it yields qualitatively physical results for rubbers.  $1/\tau$  is the flip rate of molecular segments (configuration changes), which are responsible for the viscoelastic properties of the rubber body and effectively for its internal friction. Since the flipping is a thermally activated process, it is reasonable to assume an exponential dependence of temperature in terms of an energy barrier  $\delta E$  between glassy (high frequencies) and rubber (low frequencies) regions so that  $\tau \propto \exp(\delta E/k_B T)$ .<sup>5</sup> For real rubbers the glassy to rubber transition is wide and it is characterized by a distribution of relaxation times  $\tau$  and energy barriers  $\delta E$ .<sup>5</sup>

As Eq. (1) indicates, in order to calculate the coefficient of friction the knowledge of the roughness spectrum  $C(q)$  is required. A wide variety of surfaces/interfaces are well described by a kind of roughness associated with self-affine fractal scaling.<sup>7</sup> For self-affine roughness  $C(q)$  scales as a power-law  $C(q) \propto q^{-2-2H}$  if  $q\xi \gg 1$ , and  $C(q) \propto \text{const}$  if  $q\xi \ll 1$ .<sup>7</sup> The roughness exponent  $H$  is a measure of the degree of surface irregularity,<sup>7</sup> such that small values of  $H$  characterize more jagged or irregular surfaces at short length scales ( $< \xi$ ). The self-affine scaling behavior is satisfied by the simple model<sup>8</sup>

$$C(q) = \frac{1}{2\pi} \frac{w^2 \xi^2}{(1 + a q^2 \xi^2)^{1+H}}, \quad (3)$$

with  $a = (1/2H)[1 - (1 + a Q_c^2 \xi^2)^{-H}]$  if  $0 < H < 1$  (power-law roughness), and  $a = (1/2)\ln[1 + a Q_c^2 \xi^2]$  if  $H = 0$  (logarithmic roughness).<sup>8</sup> Moreover, we have  $Q_c = \pi/a_o$  with  $a_o$  of the order of atomic dimensions, while the parameter  $w$  is the rms roughness amplitude. For other correlation models see also Refs. 9 and 10.

Our calculations were performed for  $a_o = 0.3$  nm, Poisson modulus  $\nu = 0.5$  (ignoring any weak frequency dependence),<sup>5</sup> and relatively weak applied loads  $\sigma$  so that  $E_1/\sigma \geq 100$ . Figure 1 shows calculations of the friction coefficient  $\mu_f$  for various relaxation times  $\tau$ . The maximum of  $\mu_f$  as a function of  $V$  is shifting to higher values with increasing relaxation time  $\tau$ , as well as the width of curve increases. Notably the friction coefficient  $\mu_f$  increases linearly with increasing ratio  $E_1/\sigma$  since  $\mu_f \propto E_1/\sigma$ . Indeed, if we consider Eqs. (1) and (2) we obtain

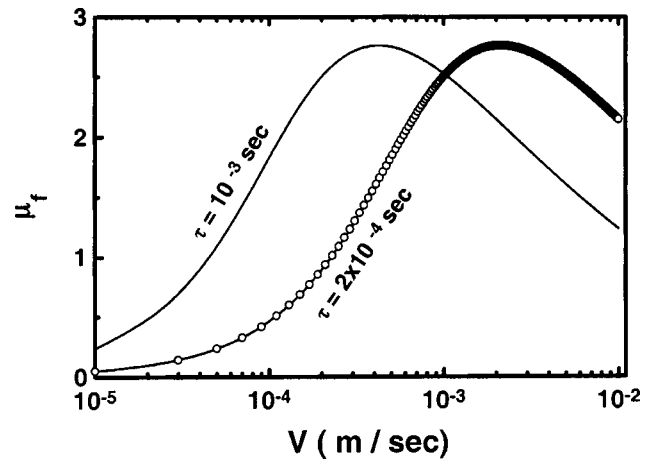


FIG. 1. Friction coefficient  $\mu_f$  vs sliding velocity  $V$  for  $E_1/\sigma = 100$ ,  $H = 0.8$ ,  $w = 2$  nm,  $\xi = 100$  nm, and various relaxation times  $\tau$  as indicated.

$$\mu_f = \frac{1}{2(1-\nu^2)} \frac{E_1}{\sigma} \int_0^{Q_c} q^3 C(q) dq \times \int_0^{2\pi} \frac{\alpha q V \tau \cos^2 \phi}{(1+\alpha)^2 + (q V \tau)^2 \cos^2 \phi} d\phi. \quad (4)$$

Since  $C(q) \propto w^2$ , the influence of the rms roughness amplitude  $w$  on the friction coefficient  $\mu_f$  is rather simple ( $\mu_f \propto w^2$ ). Therefore any complex dependence on the substrate surface roughness will arise solely from the roughness parameters  $H$  and  $\xi$  (or the ratio  $w/\xi$ ). Figure 2 shows that with increasing correlation length  $\xi$  (which leads to smoother surfaces for fixed amplitude  $w$ ), the friction coefficient  $\mu_f$  decreases. The influence is enhanced around the maximum of the velocity distribution curve. Similar is the influence of the roughness exponent  $H$  where with decreasing  $H$  or increasing surface irregularity at short roughness wavelengths ( $< \xi$ ) the friction coefficient increases (Fig. 3). The increment is highly sensitive to the value of  $H$  and as Fig. 4 indicates the effect of  $H$  is much stronger than that of the in-plane roughness correlation length  $\xi$ .

In Eq. (4) if we consider the case  $V\tau \ll \alpha/Q_c$  then we obtain the simpler form

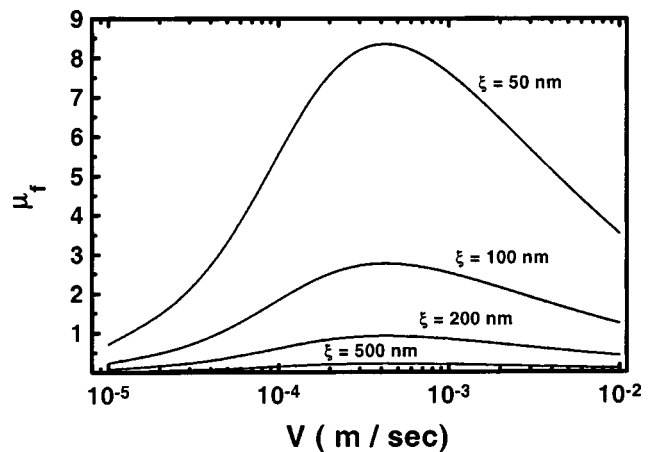


FIG. 2. Friction coefficient  $\mu_f$  vs sliding velocity  $V$  for  $E_1/\sigma = 100$ ,  $\tau = 10^{-3}$  s,  $H = 0.8$ ,  $w = 2$  nm, and various correlation lengths  $\xi$  as indicated.

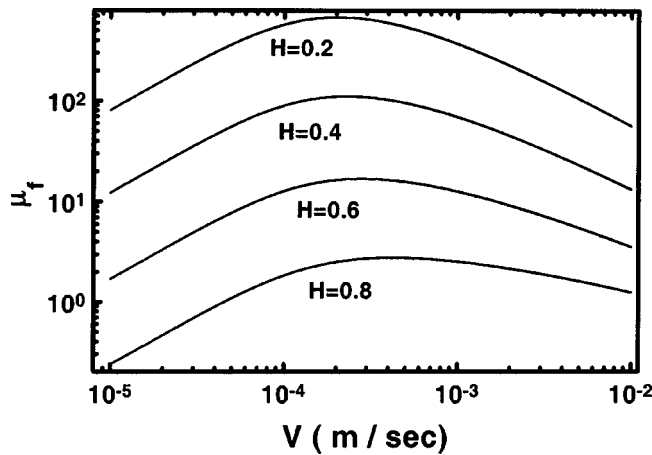


FIG. 3. Friction coefficient  $\mu_f$  vs sliding velocity  $V$  for  $E_1/\sigma=100$ ,  $\tau=10^{-3}$  s,  $w=2$  nm,  $\xi=100$  nm, and various roughness exponents  $H$  as indicated.

$$\mu_f \cong \frac{\pi\alpha}{2(1-\nu^2)(1+\alpha)^2} \frac{E_1}{\sigma} (V\tau) \int_0^{Q_c} q^4 C(q) dq, \quad (5)$$

which shows a linear dependence on the characteristic length  $V\tau$ . Clearly, Eq. (5) is valid for Figs. 2 and 3 prior to the maximum value of the velocity distribution. For the roughness exponent  $H=0$  where (Figs. 3 and 4) the coefficient of friction has the maximum value, we obtain the analytic result

$$\mu_{f(H=0)} \cong \frac{\alpha}{4(1-\nu^2)(1+\alpha)^2} \frac{E_1}{\sigma} (V\tau) w^2 \xi^2 \times \left[ \frac{Q_c^3}{3\alpha\xi^2} - \frac{Q_c}{\alpha^2\xi^4} + \frac{1}{\alpha^{5/2}\xi^5} \tan^{-1}(Q_c\xi\sqrt{\alpha}) \right]. \quad (6)$$

On the other hand we obtain also an analytic expression for the roughness exponent  $H=1$  where the friction coefficient is minimum. Indeed, we have in this case

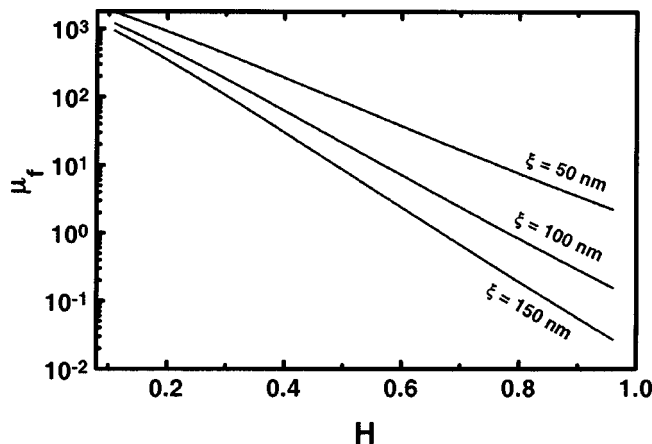


FIG. 4. Friction coefficient  $\mu_f$  vs roughness exponent  $H$  for  $V=2 \times 10^{-4}$  m/s,  $\tau=10^{-3}$  s,  $E_1/\sigma=100$ ,  $w=2$  nm, and various correlation lengths  $\xi$  as indicated.

$$\mu_{f(H=1)} \cong \frac{\alpha}{4(1-\nu^2)(1+\alpha)^2} \frac{E_1}{\sigma} (V\tau) w^2 \xi^2 \times \left[ \frac{Q_c}{\alpha^2\xi^4} + \frac{Q_c}{2\alpha^2\xi^4(1+aQ_c^2\xi^2)} - \frac{3}{2\alpha^{5/2}\xi^5} \tan^{-1}(Q_c\xi\sqrt{\alpha}) \right]. \quad (7)$$

Equations (6) and (7) set analytic physical limits of the friction coefficient so that  $\mu_{f(H=0)} \leq \mu_{f(H)} \leq \mu_{f(H=1)}$ .

At any rate our results indicate that the friction coefficient as a function of the roughness exponent  $H$  increases in agreement with intuition since the surface becomes rougher at short wavelengths ( $<\xi$ ). This is in partial disagreement with earlier findings<sup>5</sup> where the friction coefficient was found to decrease after the roughness exponent  $H$  became less than 0.2. We attribute this event to the fact that as the exponent  $H$  becomes very low and approaches 0 then the roughness becomes logarithmic which should be taken properly into account<sup>8,10</sup> by considering the whole range of lateral wavelengths from  $q\xi > 1$  to  $q\xi \ll 1$ .

### III. COEFFICIENT OF FRICTION UNDER INCOMPLETE CONTACT

Extension of these studies to the case of partial contact is also in progress in order to account for the more physical case of incomplete attachment.<sup>5</sup> Thus, if contact occurs up to a lateral length scale  $\lambda_{\text{con}} = 2\pi/q_{\text{con}}$ , Eq. (4) obtains the more complicated form

$$\mu_f = \frac{1}{2} \int_{q_{\text{con}}}^{Q_c} q^3 C(q) P(q, q_{\text{con}}) dq \times \int_0^{2\pi} \text{Im} \left[ \frac{E^*(qV\tau \cos \phi)}{(1-\nu^2)\sigma} \right] \cos \phi d\phi, \quad (8)$$

with the contact factor  $P(q, q_{\text{con}})$  given by<sup>5</sup>

$$P(q, q_{\text{con}}) = \frac{2}{\pi} \int_0^{+\infty} \frac{\sin x}{x} e^{-x^2 G(q, q_{\text{con}})} dx, \quad (9)$$

$$G(q, q_{\text{con}}) = \frac{1}{8} \int_{q_{\text{con}}}^q q^3 C(q) dq \int_0^{2\pi} \left| \frac{E^*(qV\tau \cos \phi)}{(1-\nu^2)\sigma} \right|^2 d\phi. \quad (10)$$

For low sliding velocities so that  $V\tau \ll \alpha/Q_c$  we have from Eq. (10) and Eq. (2) the simpler form for  $G(q, q_{\text{con}})$ :

$$G(q, q_{\text{con}}) \cong \frac{\pi(1+\alpha)^{-2}}{4(1-\nu^2)^2} \left( \frac{E_1}{\sigma} \right)^2 \int_{q_{\text{con}}}^q q^3 C(q) dq. \quad (11)$$

Upon substitution of Eq. (3) we obtain the analytic result

$$G(q, q_{\text{con}}) \cong \frac{(1+\alpha)^{-2}}{16(1-\nu^2)^2} \left( \frac{E_1}{\sigma} \right)^2 \frac{w^2}{a^2 \xi^2} \left[ \frac{1}{1-H} \{T_q^{1-H} - T_{\text{con}}^{1-H}\} + \frac{1}{H} \{T_q^{-H} - T_{\text{con}}^{-H}\} \right], \quad (12)$$

with  $T_q = (1 + aq^2\xi^2)$  and  $T_{\text{con}} = (1 + aq_{\text{con}}^2\xi^2)$ .

Notably, as it is shown in Ref. 5 the factor  $P(q, q_{\text{con}})$  can be well approximated by the formula  $P(q, q_{\text{con}}) = \{1$

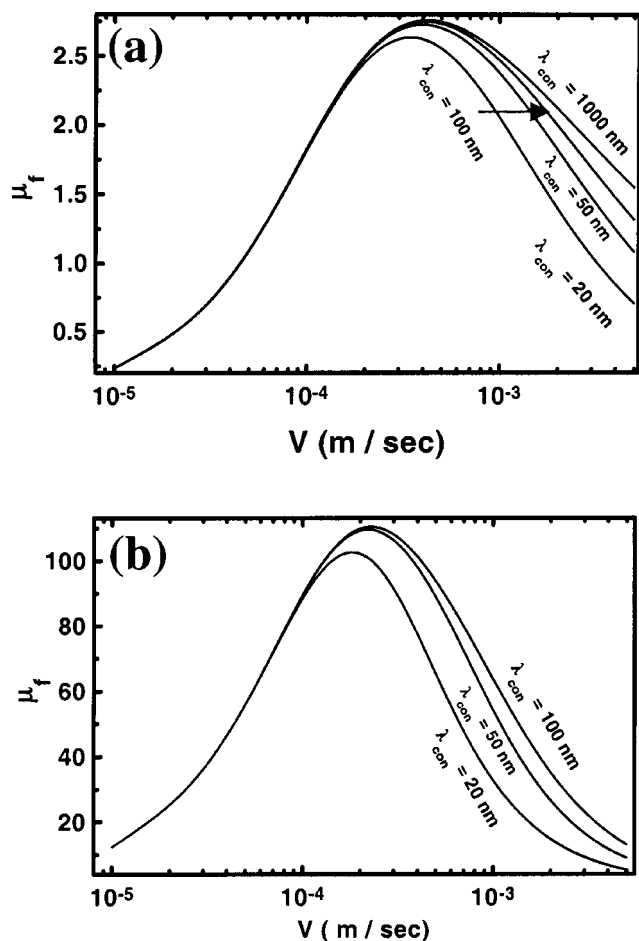


FIG. 5. Friction coefficient  $\mu_f$  vs low sliding velocity  $V$  (m/s) for  $E_1/\sigma = 100$ ,  $w = 2$  nm,  $\xi = 100$  nm,  $\tau = 10^{-3}$  s, and various contact lengths  $\lambda_{con}$  as indicated. (a)  $H = 0.8$  and (b)  $H = 0.4$ .

$+ [\pi G(q, q_{con})]^{3/2} \}^{-1/3}$  which makes calculations simpler. Calculations of the friction coefficient  $\mu_f$  for various contact lengths  $\lambda_{con}$  (and two different roughness exponents) are shown in Fig. 5. Indeed, the maximum shifts to lower velocities with decreasing contact length below the in-plane roughness correlation length ( $\lambda_{con} < \xi$ ). Moreover, the main influence of  $\lambda_{con}$  occurs at higher velocities (after the maximum) leading to lower friction coefficients especially for contact lengths  $\lambda_{con} < \xi$  and large roughness exponents ( $H > 0.5$ ).

#### IV. CONCLUSIONS

In summary, our calculations indicate that the coefficient of friction increases monotonically with decreasing roughness exponent  $H$  and at a rate that strongly increases as  $H$  becomes less than 0.5. This behavior is in agreement with the

fact that as the surface becomes rougher at short wavelengths ( $< \xi$ ) the friction increases since the rubber comes in contact with larger substrate area within sharper surface crevices. Similar is the behavior with decreasing in-plane roughness correlation length  $\xi$ . Moreover, at low sliding speeds analytic calculations were feasible for the two extreme cases of roughness exponents  $H = 0$  (logarithmic roughness<sup>8,10</sup>) and  $H = 1$  (hill-valley roughness behavior<sup>7</sup>). Finally, for conditions of incomplete contact the friction coefficient decreases below its value of complete contact for contact lengths  $\lambda_{con} < \xi$  and relatively high sliding velocities.

Although our work is theoretical, it would be interesting to test it by rubber sliding experiments on well-defined random rough surfaces. The later can vary from microns in size (produced, i.e., by scratch/wear testers) down to nanometers (i.e., polished glass surfaces<sup>5</sup> or surfaces fabricated by metal evaporation onto metallic substrates, etc.<sup>7</sup>). In addition, the adhesional contribution to rubber friction can also be studied, which was also suggested in earlier studies,<sup>5</sup> and it is expected to be significant for relatively flat surfaces ( $w/\xi \ll 1$  and  $H \approx 1$ ). Notably, the problem of rubber heating during sliding should be also addressed by solution of the heat diffusion equation<sup>5</sup> with the appropriate boundary and external conditions (i.e., substrate temperature) at the rubber/substrate interface.

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- <sup>1</sup>D. F. Moore, *The Friction and Lubrication of Elastomer* (Pergamon, Oxford, 1972); M. Barguis, *Mater. Sci. Eng.* **73**, 45 (1985); A. D. Roberts, *Rubber Chem. Technol.* **65**, 673 (1992).
- <sup>2</sup>K. A. Grosch, *Proc. R. Soc. London, Ser. A* **274**, 21 (1963).
- <sup>3</sup>J. A. Greenwood, *Fundamentals of Friction, Macroscopic and Microscopic Processes*, edited by I. L. Singer and H. M. Polack (Kluwer, Dordrecht, 1992); J. A. Greenwood and J. B. P. Williamson, *Proc. R. Soc. London, Ser. A* **295**, 300 (1966).
- <sup>4</sup>B. N. J. Persson, *Sliding Friction: Physical Principles and Applications*, 2nd ed. (Springer, Heidelberg, 2000).
- <sup>5</sup>B. N. J. Persson, *J. Chem. Phys.* **115**, 3840 (2001).
- <sup>6</sup>B. B. Mandelbrot, *The Fractal Geometry of Nature* (Freeman, New York, 1982).
- <sup>7</sup>J. Krim and G. Palasantzas, *Int. J. Mod. Phys. B* **9**, 599 (1995); Y.-P. Zhao, G.-C. Wang, and T.-M. Lu, in *Characterization of Amorphous and Crystalline Rough Surfaces-Principles and Applications, Experimental Methods in the Physical Science*, Vol. 37 (Academic Press, San Diego, California, 2001); P. Meakin, *Fractals, Scaling, and Growth Far from Equilibrium* (Cambridge University Press, Cambridge, England, 1998).
- <sup>8</sup>G. Palasantzas, *Phys. Rev. B* **48**, 14472 (1993); **49**, 5785 (1994).
- <sup>9</sup>S. K. Sinha, E. B. Sirota, S. Garoff, and H. B. Stanley, *Phys. Rev. B* **38**, 2297 (1988); H.-N. Yang and T.-M. Lu, *Phys. Rev. B* **51**, 2479 (1995); Y.-P. Zhao, G.-C. Wang, and T.-M. Lu, *Phys. Rev. B* **55**, 13938 (1997); G. Palasantzas and J. Krim, *ibid.* **48**, 2873 (1993).
- <sup>10</sup>G. Palasantzas, *Phys. Rev. E* **49**, 1740 (1994).